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The ρ 's may also be thought of as representing points.

167. Let p_1, p_2, p_3 denote the vertices of a reference triangle whose sides are of unit length and p any point in their plane. Then $|p_1=p_2p_3|, |p_2=p_3p_1|, |p_3=p_1p_2|$, and $p|p_1, p|p_2, p|p_3$ are proportional to the perpendiculars from p on the several sides of the triangle (94).

We shall consider only homogeneous equations. For, if any equation should not be homogeneous in p , all that is necessary to make it such is to introduce the factor $1=3p|p_s$ (160). Now the most general form of the equation of the second degree in trilinear coördinates is

$$a[p|p_1]^2 + b[p|p_2]^2 + c[p|p_3]^2 + 2d[p|p_2][p|p_3] + 2e[p|p_3][p|p_1] + 2f[p|p_1][p|p_2] = 0.$$

$$\text{Let } [(ap_1 + fp_2 + ep_3)p|p_1] + [(fp_1 + bp_2 + dp_3)p|p_2] + [(ep_1 + dp_2 + cp_3)p|p_3] \equiv \phi p.$$

When this value of ϕp is substituted in the preceding equation it reduces to $p|\phi p = 0$. Hence $p|\phi p = 0$ is the equation for all quadric curves whether central or non-central. Had quadriplaner coördinates been employed and the corresponding expressions constructed, an equation would have resulted representing any and all quadric surfaces. The same method may be used in getting the equation of the quadric in n -dimensional space.

REMARK.—The introduction of the ϕ function from Hamilton into the *Ausdehnungslehre* is due to Professor Hyde. (*Directional Calculus*, page 103). He shows that point analysis gives a means of changing the ordinary Cartesian equations into equations analogous to those of trilinear coördinates and then of generalizing the application of the equation $p|\phi p = 0$ to include the case of quadrics, central and non-central.

[To be Concluded.]



DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

132. Proposed by WILLIAM SYMMONDS, A.M., Professor of Mathematics, Santa Rosa College, Sebastopol, Cal.

A road 60 feet wide crosses a square acre of land. The west line of the road passes through the southwest corner of the land, while the east line of the former passes through the northeast corner of the latter. What fraction of the land is included in the road?

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; D. G. DORRANCE, Jr., Camden, N. Y.; J. SCHEFFER, A. M., Hagerstown, Md.; and G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $ACBD$ =the square acre, $AGBE$ =the road, and EF perpendicular to BG , $=60$ feet.

Put $a=AC=BC$, and $b=EF$.

Let $x=BE$, and $y=BG$.

Then $a-x=GC$, ax =area of road, and x/a =fractional part of square acre included in the road.

From the similar right triangles EFB and BCG , we have $x:y=b:a$; whence $y=ax/b$.

Also from right triangle BCG , $y=\sqrt{a^2+(a-x)^2}$.

$$\therefore \frac{a^2x^2}{b^2}=2a^2-2ax+x^2.$$

$$\text{Whence } x=\frac{ab[-b\pm\sqrt{(2a^2-b^2)}]}{a^2-b^2}, \text{ and } \frac{x}{a}=\frac{b[-b\pm\sqrt{(2a^2-b^2)}]}{a^2-b^2}.$$

Substituting the numerical values, $b=60$ and $a^2=43560$ [=the area of an acre in square feet], we obtain $x/a=344$.

QUERY. When $a=b$, what is the value of $\frac{ab[-b\pm\sqrt{(2a^2-b^2)}]}{a^2-b^2}$? GRUBER.

ANSWER. By differentiating both numerator and denominator with respect to b and then reducing we find the value of the expression to be equal to a , for either the $+$ or $-$ sign. It may also be shown as follows:

$$\begin{aligned} \frac{ab[\pm\sqrt{(2a^2-b^2)}-b]}{a^2-b^2} &= \frac{ab[\pm\sqrt{(2a^2-b^2)}-b]}{\frac{1}{2}(2a^2-b^2-b^2)} \\ &= \frac{2ab[\pm\sqrt{(2a^2-b^2)}-b]}{[\sqrt{(2a^2-b^2)}+b][\sqrt{(2a^2-b^2)}-b]} = \frac{2ab}{\sqrt{(2a^2-b^2)}+b} \text{ or } -\frac{2ab}{\sqrt{(2a^2-b^2)}-b} \\ &= a \text{ or } -\infty. \end{aligned}$$

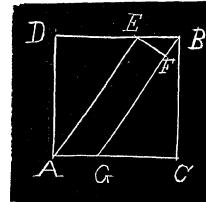
These values might have been found by making the assumption that $a=b$ in the equation from which the expression arose.

If, however, we write the denominator of the expression for the roots $2a^2-b^2-a^2=[\sqrt{(2a^2-b^2)}+a][\sqrt{(2a^2-b^2)}-a]$, and then divide $+\sqrt{(2a^2-b^2)}-b$ by $[\sqrt{(2a^2-b^2)}-a]$ we get

$$1 + \frac{a-b}{\sqrt{(2a^2-b^2)}} + \frac{a(a-b)}{(2a^2-b^2)^{\frac{1}{2}}} + \frac{a^2(a-b)}{(2a^2-b^2)^{\frac{3}{2}}} + \text{etc.},$$

and the value of the root is

$$\frac{ab}{[\sqrt{(2a^2-b^2)}+a]} \left(1 + \frac{a-b}{(2a^2-b^2)^{\frac{1}{2}}} + \frac{a(a-b)}{(2a^2-b^2)} + \frac{a^2(a-b)}{(2a^2-b^2)^{\frac{3}{2}}} + \text{etc.} \right)$$



While each of these terms after the first, approach 0 as $a=b$, making it appear that from this view one root is $\frac{1}{2}a$ instead of a , as found above; yet by writing the series as follows:

$$1 + (a-b) \left[\frac{a}{\sqrt{(2a^2-b^2)}} + \frac{a^2}{(2a^2-b^2)} + \frac{a^3}{(2a^2-b^2)^{\frac{3}{2}}} + \text{etc.} \right],$$

it is seen that this factor takes the form, in the limit, $1+0 \times \infty$; and, therefore, this method is no more capable of yielding a determinate result than is the original expression. **EDITOR F.**

ALGEBRA.

108. Proposed by GEORGE LILLEY, Ph.D., LL.D., Professor of Mathematics, State University, Eugene, Or.

A gave two notes; one for a dollars at m per cent., and the other for b dollars at n per cent. annual interest. He is to make a monthly payment of c dollars. How much must be endorsed on each note in order to pay them off at the same time? What must be the endorsement on each if $a=1900$, $b=1800$, $m=6$, $n=7$, and $c=25$.

[This problem is the same as No. 86, Miscellaneous. See the solutions in that department.]

109. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

$$\frac{1}{\sqrt[3]{(x+1)}} + \frac{1}{\sqrt[3]{(x-1)}} = \frac{1}{\sqrt[3]{(x^2-1)}}; \text{ find value of } x \text{ satisfying the equation.}$$

Solution by the PROPOSER.

This problem was proposed for the purpose of explaining the singular fact (singular to those who do not possess more than a mechanical knowledge of algebra) that the value of the unknown does not satisfy the original equation.

By transposing and factoring, the original equation may be written

$$\frac{1}{\sqrt[x^2-1]}[\sqrt[x-1]+1-\sqrt[x+1]-1]=0,$$

which is equivalent to the system of equations,

The solution of A gives $x=\infty$, which value satisfies the original equation.

From B we have $\sqrt{x+1} = 1 - \sqrt{x-1}$. Squaring both sides, transposing and combining, we have $1 = -2\sqrt{x-1}$.

From this last, by squaring, we obtain $1=4(x-1)$, from which we find,